Factoring Trinomials $ax^2 + bx + c$

Lesson 7.8

Essential Question How can you factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

In Section 7.7, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial.

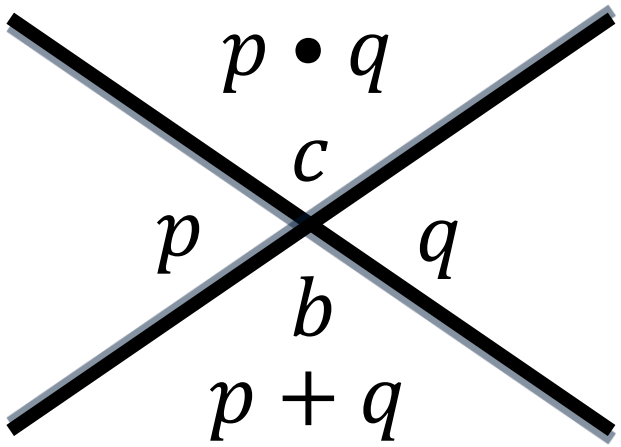
EXAMPLE 1 Factoring Out the GCF

Factor $5x^2 + 15x + 10$.

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$$5x^2 + 15x + 10 = 5(x^2 + 3x + 2)$$

The Magic X



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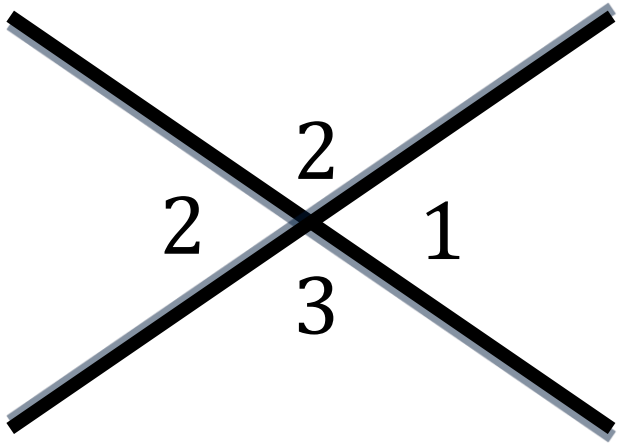


$$= 5(x + 1)(x + 2)$$

When there is no GCF, factor using the Bottoms Up Method.

Example: $5x^2 + 17x - 12$

Step 1: Determine ac. $5 \bullet (-12) = -60$

Step 2: Write $x^2+bx+ac$ $x^2 + 17x - 60$

Step 3: Find two factors of ac that add to b. $(x - 3)(x + 20)$

Step 4: Divide by a. $(x - \frac{3}{5})(x + \frac{20}{5})$

Step 5: Reduce. $(x - \frac{3}{5})(x + \frac{4}{1})$

Step 6: Bring Bottoms Up. $(5x - 3)(x + 4)$

Notice: Denominators become leading coefficients and numerators become the factors.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When ac Is Positive

a. Factor $4x^2 + 13x + 3$.

$4 \cdot 3 = 12$

$x^2 + 13x + 12$

$(x + 12)(x + 1)$

$(x + \frac{12}{4})(x + \frac{1}{4})$

$(x + 3)(x + \frac{1}{4})$

$(x + 3)(4x + 1)$

b. Factor $3x^2 - 7x + 2$.

$3 \cdot 2 = 6$

$x^2 - 7x + 6$

$(x - 6)(x - 1)$

$\left(x - \frac{6}{3}\right)\left(x - \frac{1}{3}\right)$

$(x - 2)\left(x - \frac{1}{3}\right)$

$(x - 2)(3x - 1)$

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Factor $2x^2 - 5x - 7$.

$$2 \cdot (-7) = -14$$

$$x^2 - 5x - 14$$

$$(x + 2)(x - 7)$$

$$(x + \frac{2}{2})(x - \frac{7}{2})$$

$$(x + 1)(x - \frac{7}{2})$$

$$(x + 1)(2x - 7)$$

On Your Own

Factor the polynomial.

1. $8x^2 - 56x + 48$

   $8(x-1)(x-6)$

2. $2x^2 + 11x + 5$

   $(2x+1)(x+5)$

3. $2x^2 - 7x + 5$

   $(2x-5)(x-1)$

4. $3x^2 - 14x + 8$

   $(3x-2)(x-4)$

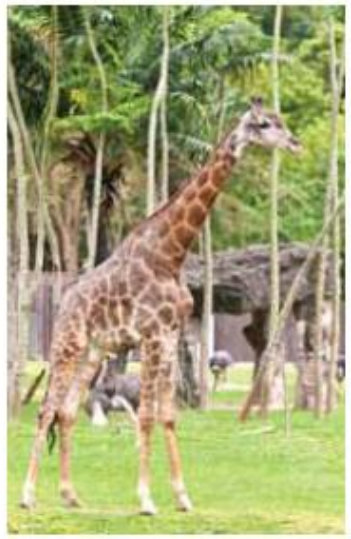
5. $6x^2 + x - 12$

   $(3x-4)(2x+3)$

6. $4x^2 - 19x - 5$

   $(4x+1)(x-5)$

Real-Life Application



The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. How wide is the reserve?

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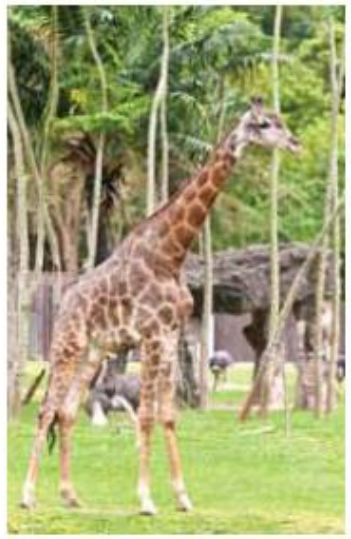

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Write an equation that represents the area of the reserve. Then solve by factoring. Let w represent the width. Then $2w + 1$ represents the length.

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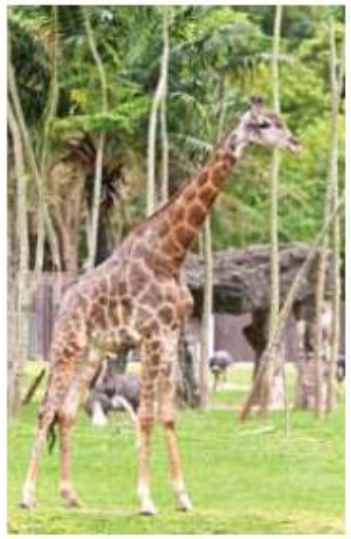

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$$w(2w + 1) = 55$$

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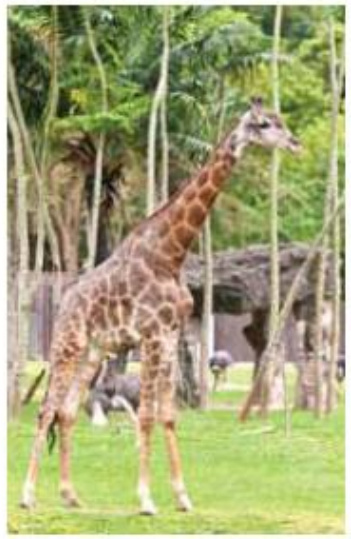

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$$w(2w + 1) = 55$$

$$2w^2 + w - 55 = 0$$

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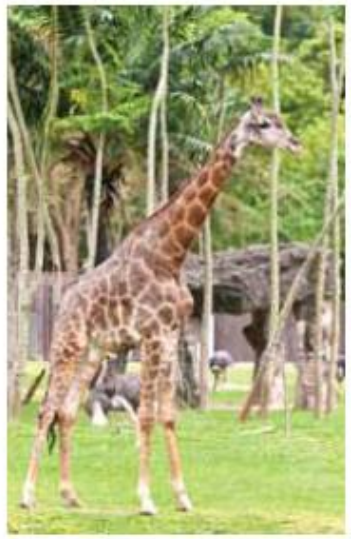

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$$w(2w + 1) = 55$$

$$2w^2 + w - 55 = 0$$

$$(w - 5)(2w + 11) = 0$$

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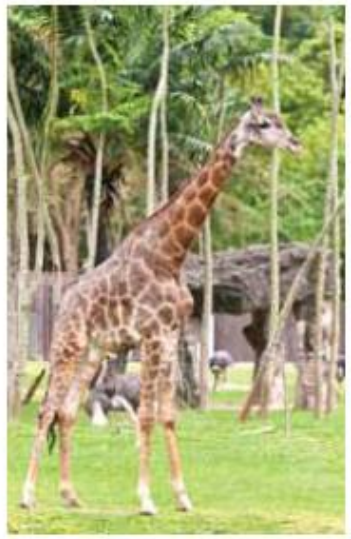

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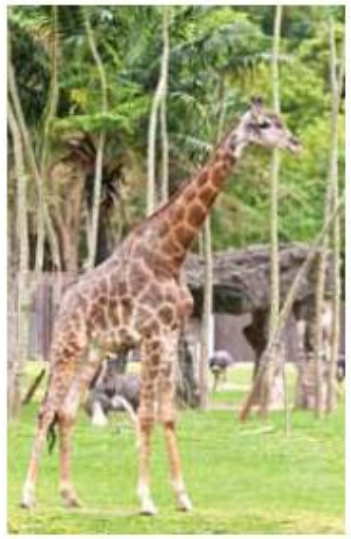

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. How wide is the reserve? The reserve is 5 miles wide.

Write an equation that represents the area of the reserve. Then solve by factoring. Let w represent the width. Then $2w + 1$ represents the length.

$$w(2w + 1) = 55$$

$$2w^2 + w - 55 = 0$$

$$(w - 5)(2w + 11) = 0$$

$$w - 5 = 0 \quad or \quad 2w + 11 = 0$$

$$w = 5 \quad or \quad w = -\frac{11}{2}$$